

MATH 1A – QUIZ 1

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Name: _____

Instructions: You have 10 minutes to do this quiz, for a total of 10 points. **Show your work, unless otherwise specified!** Good luck, and may πm be with you!

(1) (3 points) Find the domain of $f(x) = \cos\left(\frac{1}{x}\right)\sqrt{(x-3)^2 - 4}$ We want:

- 1) $x \neq 0$ (because we want the denominator of the fraction $\frac{1}{x}$ to be nonzero)
- 2) $(x-3)^2 - 4 \geq 0$ (because we want the number under the square root to be ≥ 0).

Solving this, we get: $(x-3)^2 \geq 4$, that is $x-3 \leq -2$ or $x-3 \geq 2$, so
 $x \leq 1$ or $x \geq 5$

- 3) Combining, we get: $x \neq 0$ and $(x \leq 1$ or $x \geq 5)$

Answer: $\text{Dom}(f) = (-\infty, 0) \cup (0, 1] \cup [5, \infty)$ ¹

(2) (2 points) Find the range of $f(x) = 3 \sin(x) + 2$. Here you do **NOT** have to show any work.

You could either draw the graph of f , or do it algebraically:

$$\begin{aligned} -1 &\leq \sin(x) \leq 1 \\ -3 &\leq 3 \sin(x) \leq 3 \\ -1 &\leq 3 \sin(x) + 2 \leq 5 \end{aligned}$$

Hence $\text{Ran}(f) = [-1, 5]$

Date: Friday, September 6th, 2013.

¹which you can also write as 'the set of x such that $x < 0$ or $0 < x \leq 1$ or $x \geq 5$ '

- (3) (2 points) Find $f \circ f$ (f composed with f), where $f(x) = \frac{1}{x+1}$. Write your answer in the form of a fraction, i.e. $\frac{ax+b}{cx+d}$, where a, b, c, d are integers.

$$(f \circ f)(x) = f(f(x)) = f\left(\frac{1}{x+1}\right) = \frac{1}{\frac{1}{x+1} + 1} = \frac{1}{\frac{1+(x+1)}{x+1}} = \frac{1}{\frac{x+2}{x+1}} = \frac{x+1}{x+2}$$

- (4) (3 points) Explain **in words** how to obtain the graph of $f(x) = 2 \sin(-x + 3) + 4$ from the graph of $y = \sin(x)$. You do **not** have to draw any graphs!

Note: The following vocabulary may be useful: Stretch/Compress horizontally/vertically by a factor of \dots , shift up/down/left/right, Flip across the x/y -axis.

Note: There are many answers to this problem, here are my two favorite ones:

Answer 1 (horizontal, then vertical): First we start with the graph of $y = \sin(x)$, and then we:

- 1) Shift the resulting graph left by 3 units (to obtain $\sin(x + 3)$)
- 2) Flip the resulting graph (horizontally) across the y -axis (to obtain $\sin(-x + 3)$)
- 3) Stretch the resulting graph vertically by a factor of 2 (to obtain $2 \sin(-x + 3)$)
- 4) Shift the resulting graph up by 4 units (to obtain $2 \sin(-x + 3) + 4$)

Answer 2 (vertical, then horizontal): First we start with the graph of $y = \sin(x)$, and then we:

- 1) Stretch the resulting graph vertically by a factor of 2 (to obtain $2 \sin(x)$)
- 2) Shift the resulting graph up by 4 units (to obtain $2 \sin(x) + 4$)
- 3) Shift the resulting graph left by 3 units (to obtain $2 \sin(x + 3) + 4$)
- 4) Flip the resulting graph (horizontally) across the y -axis (to obtain $2 \sin(-x + 3) + 4$)